ON THE DIFFERENTIAL APPROACH TO RADIATIVE HEAT TRANSFER

P. K. Konakov

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In [1] the author solved the problems of radiative heat transfer in plane, cylindrical, and spherical layers of a gray gas. The solutions were based on the differential representation of the transfer of radiative energy in a gray gas and on boundary conditions based on the assumption of radiative equilibrium in the boundary layer of the gas.

Adrianov and Polyak have proposed new boundary conditions [4], which lead to contradictory results and must, therefore, be erroneous. In a later publication [5] these authors discuss the question of the boundary conditions of the differential equations of radiative heat transfer. The erroneous assumptions in that paper require critical discussion.

Adrianov and Polyak consider the problem of radiative heat transfer in a plane layer of a gray gas in local radiative equilibrium. This state is determined by the equation

$$\frac{d}{dx}\left(D\,\frac{du}{dx}\right)=0\,, \tag{1}$$

$$E_{\text{T}}-cU/4=0. \tag{2}$$

The authors assume

$$D=c/3k,$$

so that for constant k

$$d^2U/dx^2=0.$$

Using the boundary conditions formulated by Adrianov and Polyak, one can write

$$q=\left(\frac{1}{A_1}-\frac{1}{2}\right)^{-1}(E_{01}-cU_1/4),\;\; q=\left(\frac{1}{A_2}-\frac{1}{2}\right)^{-1}\left(\frac{cU_2}{4}-E_{02}\right). \tag{3}$$

The equation $d^2U/dx^2 = 0$ was integrated between the limits $x = 0$ and $x = \delta$, yielding

$$U=-\frac{U_1-U_2}{\delta}x+U_1.$$

Hence

$$q=\frac{1}{3k\,\delta/4}\left(\frac{cU_1}{4}-\frac{cU_2}{4}\right). \tag{4}$$

Solving (3) and (4) simultaneously, Adrianov and Polyak obtain the equation

$$q=(E_{01}-E_{02})\left(\frac{1}{A_1}+\frac{1}{A_2}-1+\frac{3}{4}\,k\,\delta\right)^{-1}.$$

This shows that, in solving the elementary problem under consideration, Adrianov and Polyak completely ignore the condition of local radiative equilibrium, represented by (2).

Taking this condition into account, one obtains at the bounding planes of the layer

$$E_{01}-\frac{cU_1}{4}=0,\;\;\frac{cU_2}{4}-E_{02}=0,$$

which leads to $q = 0$ at the bounding planes and

$$q = (E_{01} - E_{02})(3k\delta/4)^{-1}$$

inside the layer.

This contradictory result is due to the incorrectly posed boundary conditions, as the authors allow radiative nonequilibrium at the bounding planes of an equilibrium layer. This is inconsistent with (2) and leads to the above contradiction.

The contradiction can be eliminated by deriving the equation for q from the assumption of radiative equilibrium of a boundary layer of thickness $1/k$ [6]. The formulation of the radiative equilibrium of the boundary layer requires the application of Buguer's law. Thus, Adrianov and Polyak's remark that the boundary conditions derived by the author contradict Buguer's law is incorrect.

The incorrectness of the boundary condition in [5] is also indicated by the fact that the solution of the radiative heat transfer problem in cylindrical and spherical gas layers does not, under these conditions, lead in the limit to Christiansen's formula.

Further, Adrianov and Polyak define the diffusion coefficient of radiative energy transfer by the expression $D = c/3k$, which is incorrect. Simple calculation shows that for a plane layer of a gray gas $D = c/4k$. The derivation of this formula can be found in several papers [6, 7].

Now as regards the comparison between my solution and the "exact" solution of Hottel [2].

A study of Hottel's graph [2] shows that for values $k\delta \geq 6$ my solution practically coincides with Hottel's "exact" solution. Thus Adrianov and Polyak's remark that for large $k\delta$ my solution leads to results 25% lower than Hottel's "exact" values is incorrect.

In conclusion, I would like to remark that there is no sufficient foundation for regarding Hottel's solution as absolutely exact, as the mathematical formulation of the problem considered by Hottel is by no means clear. It can be said only that correct application of the differential and integral approaches to radiative heat transfer should lead to identical results.

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Institute of Railroad Engineers,
Moscow

HEAT TRANSFER ON A NONISOTHERMAL FLAT PLATE WITH A LAMINAR BOUNDARY LAYER

D. A. Labuntsov

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Reference [1] contains the very interesting results of an experimental investigation of heat transfer in longitudinal flow over a nonisothermal flat plate. In particular, data are given for the case of an initial adiabatic section followed by a region with approximately constant heat flux. Under these conditions (which had not been previously examined experimentally) the unheated length was observed to have an appreciable effect on heat transfer in the presence of a laminar boundary layer.

This effect is described in [1] by the empirical relation: